SOLID MECHANICS LABORATORY TM 9334-018

BALANCED STRESSES IN POST-YIELDED MULTI-MATERIAL STRUCTURAL JOINTS

BY

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JULY 5, 1969

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PHILADELPHIA, PA.

July 5, 1969

For

FINITE ELEMENT APPLICATION SYMPOSIUM

Vanderbilt University
Nashville, Tennessee
November, 1969

NAS-3-10610

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1.0 SUMMARY

Under very severe mechanical and thermal environments, structural joints yield locally and cause a redistribution of stresses within the various elements in the joint. Present day codes permit this type of local yielding and many such joints in actual operation have been found to be structurally adequate for specific applications.

This paper presents a unique application of the finite element method in solving the stresses in an axisymmetric post-yielded tongue-and-groove cylindrical joint composed of a tantalum tongue, brazed to a stainless steel groove by the use of a cobalt base alloy (similar to L605). The stress distribution in the mini-brazed areas is also investigated. In all cases, bi-linear stress-strain relations are established to facilitate the solution of the problem.

Iso-stress maps in the joint are generated giving the structural engineer a bird's eye view of the stress variations, similar to those from photoelasticity tests. The study concludes that, owing to the strain compatibility relations, the stresses in the post-yielded multi-material joint are "balanced"; i.e., their peak values from the elasticity solutions are significantly reduced. Furthermore, the bi-linear analyses under the thermal loads have been found to converge quickly, limiting to only one or two iterations.

Based on procedure established, further applications may be extended to determining localized stresses in repair welds, such as welds with porosities, fracture growths, post-yielded riveted and bolted joints; many of which problems have been hitherto extremely difficult to solve theoretically by the structures engineers.

2.0 INTRODUCTION

Structural joints fabricated from bolted, riveted, welded or brazed constructions, are known to have areas of localized yielding when subjected to large applied forces. Technical approach, thus far in analyzing these types of joints, has been the use of ductile materials and the allowance of local stretching and yielding, thus permitting a conventional elasticity analysis. In many instances, because of the inability to theoretically predict the localized high stresses, structure engineers have adopted the attitude of being highly conservative in their design and left the final configuration to be determined after numerous, tedious and costly experiments.

In modern lightweight structures, subjected to extremely severe thermal environments, development of advanced analytical technique in stress mapping the joint and in understanding the plastic behavior of the elements in the joint becomes increasingly important. In large rocket motor cases and in liquid metal nuclear power plants where welded and brazed joints are frequently used, the introduction of additional non-parent materials such as welds in the joint complicates further the interaction of the elements in the joint.

The purpose of this paper is to present a unique application of the finite element method in solving the elastic and inelastic stresses in a typical multi-material structural joint subjected to very severe thermal environment. Because of the multi-material factor, together with the temperature-dependent strain-dependent properties, the finite-element method was found to be highly suitable for performing this type of stress analysis.

3.0 TECHNICAL APPROACH

3.1 Finite-Element Model

The joint considered consists of an axisymmetric cylindrical joint with the longitudinal cross-section made up of a conventional tongue-and-groove type. Three materials are considered; the tantalum tongue, the stainless steel groove, and the brazed material represented by cobalt-based L605 alloy. The joint is modelled to approximately 520 finite cylindrical elements (Figure 1) each having a wall thickness of 0.010 inch. In order to define more accurately the stress-raising effects at the discontinuous corners at the juncture of the tongue-and-groove, the element mesh is made finer at these locations as indicated in Figure 1.

3.2 Finite-Element Computer Program

The computer program, designated as ORTHOSAFE and used in the analysis, is a system of computer programs for the general thermostructural analysis of orthotropic elastic bodies, and is based on the finite element direct stiffness approach (Refs. 1 through 4). The program has the capability of computing displacements, strains, and stresses throughout the cross-section of either an axisymmetric or a planar body. The continuous structure to be analyzed is idealized into a gridwork of discrete elements, interconnected at nodes, or nodal point circles.

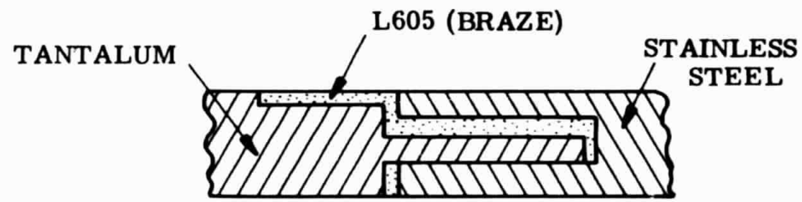
For the axisymmetric case, the element used is a ring element of quadrilateral cross-section. Within the structure, the element is represented by its four corner nodes, or in the case of the axisymmetric problem, nodal circles. At each of the nodes or nodal circles, there are two generalized forces and displacements in the radial and axial directions. For a general element, an elemental stiffness matrix is derived which relates these nodal forces and displacements. The derivation of the elemental stiffness matrix is based on an assumed linear displacement field within the element.

The bi-linear option of the program was formulated* to generalize the uniaxial data to a three-dimensional formulation so that design evaluation of bodies which are subjected to more general states of stress, than uniaxial, can be conducted.

Using the material parameters (E_o, n, f_1), defined as in Figure 2, the (effective) modulus for any value of stress may be formulated as follows:

*Per T. McDonough, Ref. (4).

STATION	Z (in.)
A	2.1525
B	2.2500
C	2.3225



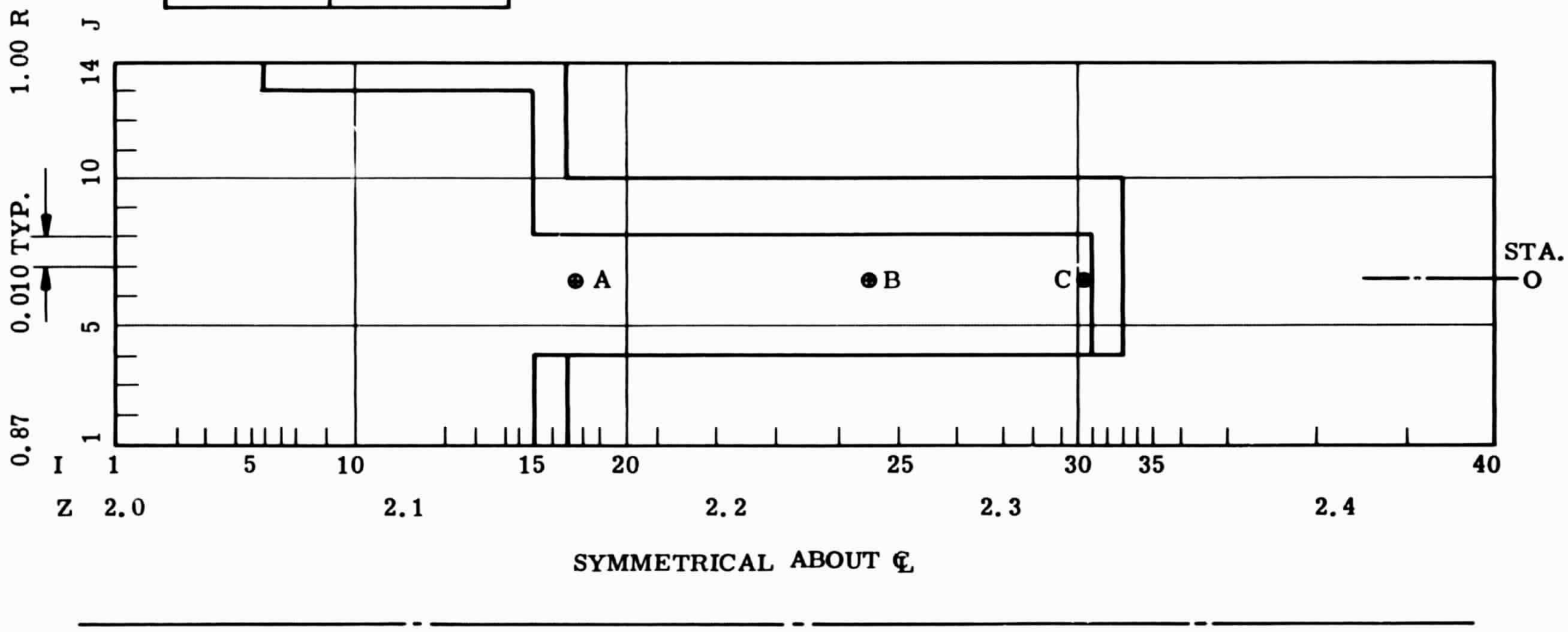


Figure 1. Finite-element Model in Multi-material Joint

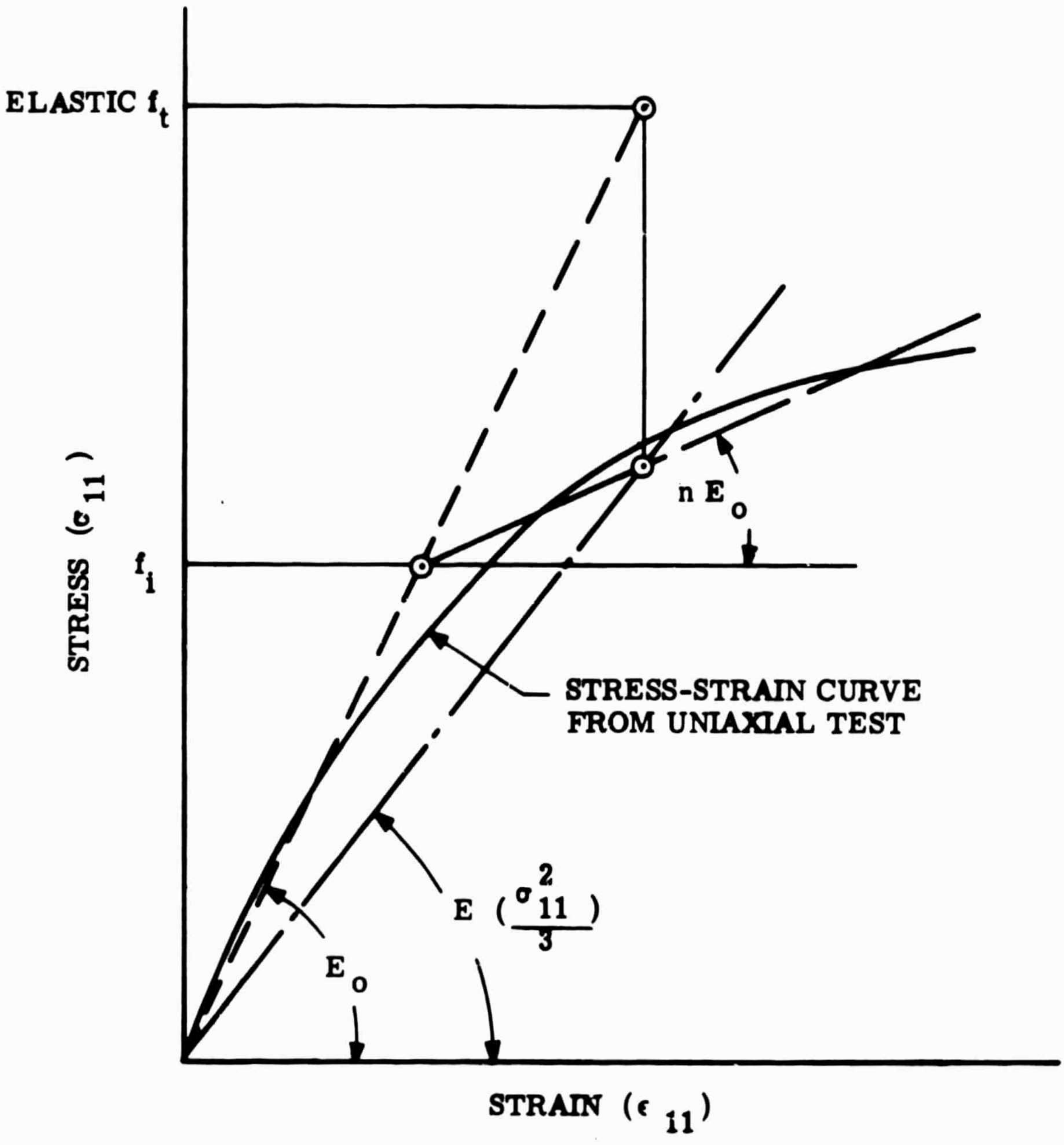


Figure 2. Bi-linear Stress-strain Approximations

$$
E\left(\frac{\sigma_{11}^2}{3}\right) = \begin{cases} E_o & \text{for } \frac{\sigma_{11}^2}{3} < \frac{f_i^2}{3} \\ \text{or} & \\ \dfrac{n\,E_o}{1 - (1-n)\dfrac{f_i}{\sigma_{11}}} & \text{for } \frac{\sigma_{11}^2}{3} > \frac{f_i^2}{3} \end{cases}
$$

The function of $E(J_2)$ in which J_2 is defined as second invariant of $\bar{\sigma}_{ij}$ for an arbitrary state of stress can be chosen to agree with the uniaxial stress data as follows:

$$
E(J_2) = \begin{cases} E_o & \text{for } J_2 < \frac{f_i^2}{3} \\ \text{or} & \\ \dfrac{n\,E_o}{1 - (1-n)\dfrac{f_i}{\sqrt{3J_2}}} & \text{for } J_2 > \frac{f_i^2}{3} \end{cases}
$$

The choice as given above is based on the prediction that plastic flow initiates in a ductile material when the effective stress, $\bar{\sigma}$, reaches the value of the uniaxial yield strength of the material in tension. It must be pointed out this theorem has been substantiated to a large degree in an experimental program using a large number of sub- and full-scale pressure vessels tested to destruction (Ref. 5).

The solution of the bi-linear problem is therefore based on successive iteration (Ref. 3) using technique restricted to linear elastic behavior. The procedure is contingent on the belief that strain state is not as devious as the stress state at the end of each iteration; therefore, the values of predicted strain stage are used to compute the effective moduli from the bi-linear stress-strain relations which are then used for the next iteration.

As will be illustrated later, the computer program includes a flexible graphic output capability which greatly aids the user in inputting data and in interpreting the results of the analysis.

4.0 ILLUSTRATIVE EXAMPLE

4.1 Problem Description

The cylindrical tongue-and-groove joint investigated consists of a tantalum tongue, a stainless steel 316 groove and a brazed material having material properties very similar to the cobalt base alloy L605. For the purpose of illustration, the stresses in question are the residual stresses in the joint, at the room temperature, due to a cool-down differential temperature of 1800°F. The stress-free state was assumed at the high temperature. As shown in Figure 1, the geometry of the cylinder and the various lengths and thicknesses of the different materials are given. The following elastic properties of the materials at room temperature are used in the analysis:

Material	Thermal Expansion $\frac{\Delta L}{L}$ in./in.	Modulus of Elasticity (psi)	Poisson's Ratio
Tantalum	2.5×10^{-4}	27.5×10^{6}	0.35
SS 316	8.2×10^{-4}	28.6×10^{6}	0.28
L605	6.6×10^{-4}	33.6×10^{6}	0.3

The bi-linear stress-strain behavior of the tri-metals at room temperature is shown in Figure 3.

4.2 Results of Investigations

The elasticity analysis was performed first. The stress maps for hoop, axial, and maximum principal stresses in the plane of radial (R) and axial (Z) directions are shown in Figures 4 through 6, respectively. Due to the severe thermal condition, extremely high stresses, well-above the yield strengths of the material, are found in the vicinity of the tips of tongue and the groove. This indicates that many elements in the multi-material joint have yielded, creating a new strain compatibility situation. The chart shown in Figure 7 is one of the outputs from the computer program giving the results of the elasticity analysis. Out of the 520 elements, 368 elements have been subjected to stresses higher than the yield strengths.

The solution now proceeds to the first bi-linear iteration, using the stress-strain relations of the individual material in the E_0 or nE_0 regions (Figure 3) dependent on the state of stresses from the elasticity solution. Similar stress maps for the first iteration are shown in Figures 8 through 10. Again, the stresses are scanned and the conditions of yielding are established for various elements as done before. The results of the second iteration are given in Figures 11 through 13.

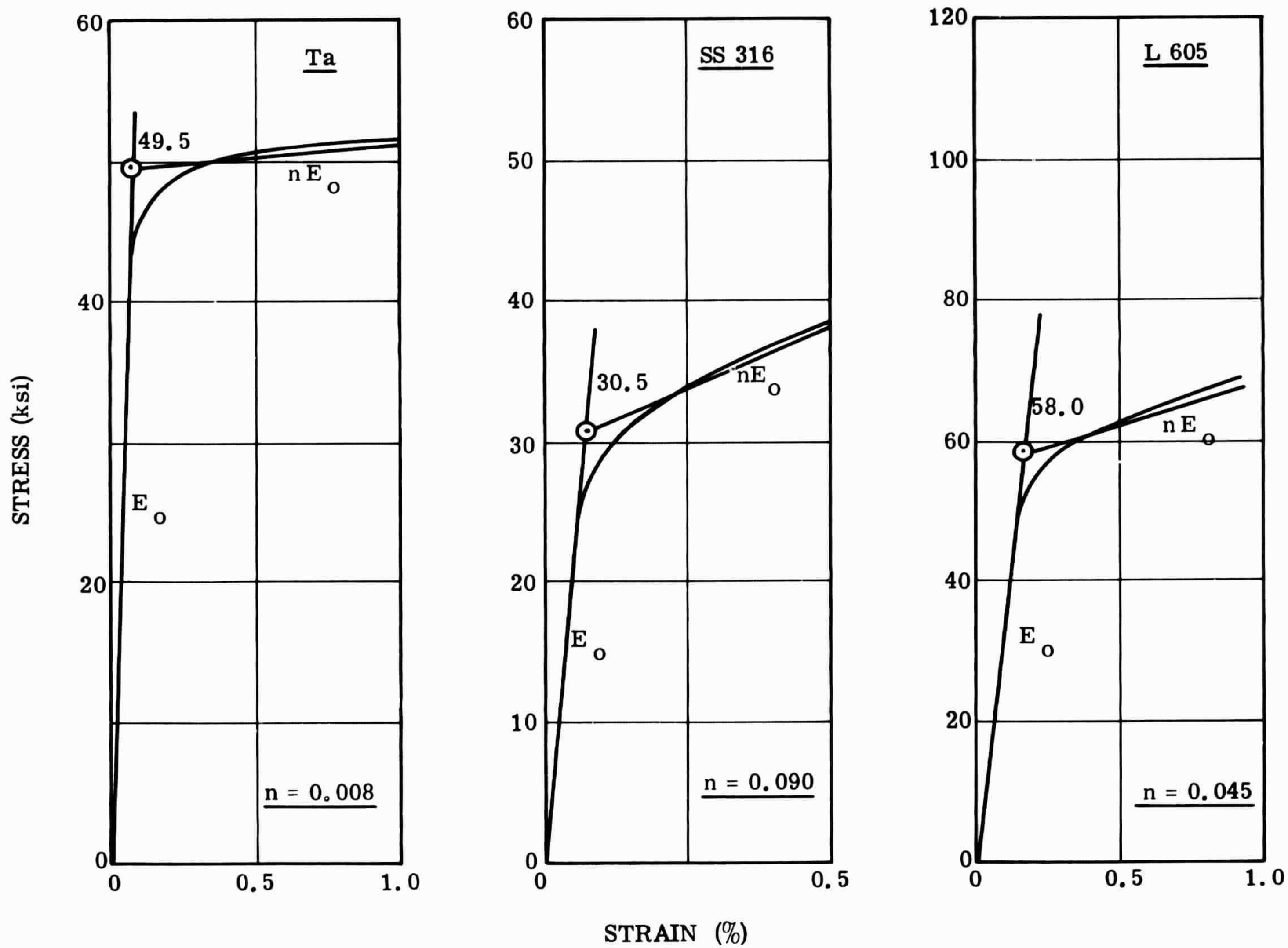


Figure 3. Bi-linear Stress-strain Relations of Joint Materials

The variations of stresses are plotted in the lengthwise direction at Station O and also across the wall thicknesses of the joint at Stations A, B and C (see Figure 1 for locations). These stations are found to yield critical stresses due to their locations in the highly discontinuous sectors of the joint. The variations for hoop stresses are shown in Figures 14 through 17 and for axial stresses in Figures 18 through 21. In all cases, the results for the bi-linear iterations 1 and 2 are superposed on the elastic plots for comparison purposes.

5.0 CONCLUSIONS

The following conclusions can be drawn from the results of this study:

1. Stresses in typical structural joints composed of multi-material and subjected to severe thermal environment can be conveniently analyzed by the finite-element technique. The solution is powerful in that it can handle mini-areas such as in the weld for brazed material as well as the finite variations of material properties either because of difference in material or because of temperature-dependent conditions.

2. The iso-stress maps generated from the finite-element analyses gave the structural engineer a bird's eye view of the stress variations, similar to those from experimental photoelasticity.

3. Because of the strain compatibility relations, the stresses in the post-yielded multi-material joint are "balanced"; i.e., their peak values from elasticity solutions are drastically reduced. These reduced values enable the engineer to predict realistically the stresses in the joint and to design a multi-material joint under optimized variable straining conditions.

4. The convergence of the bi-linear analyses under the thermal loads has been found to be quick, limiting to one or two iterations.

Although the example given was for illustration of residual stresses, the procedure has been successfully applied in determining operating stresses and in studying the cyclic behavior of the joint. Further applications of the procedure can be extended to determine localized stresses in repair welds, welds with porosities, fracture growths, post-yielded riveted or bolted joints, many of which problems have been hitherto extremely difficult to solve theoretically by the structures engineers.

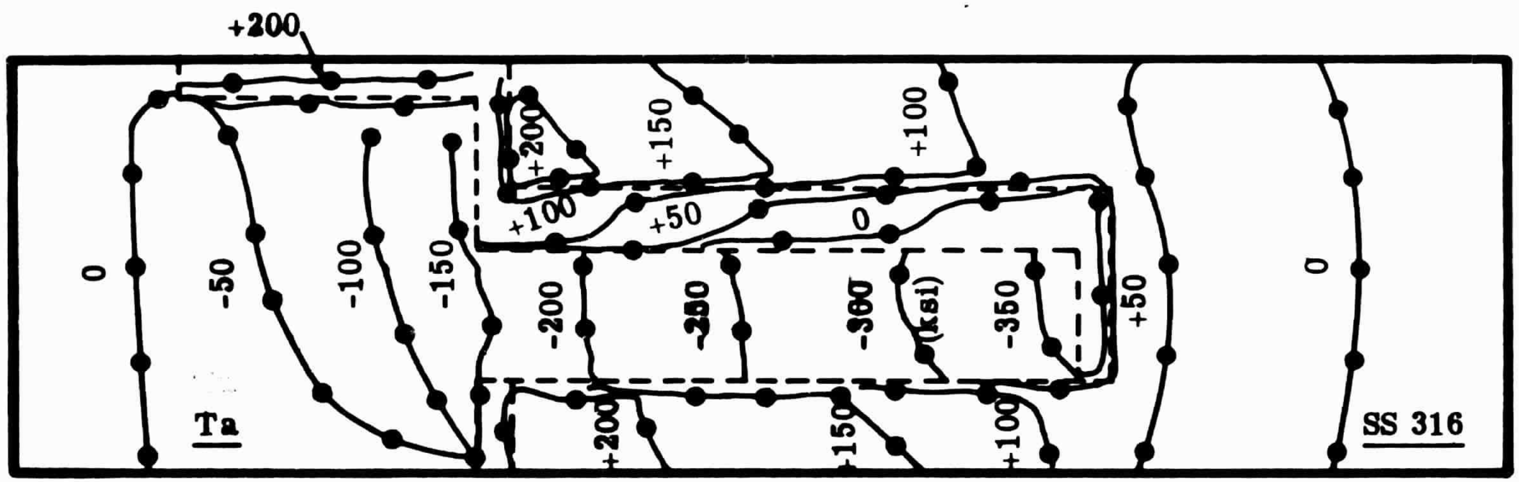


Figure 4. Iso-stress Map in Multi-material Joint Residual Hoop Stresses (Elastic) Due Cool-down

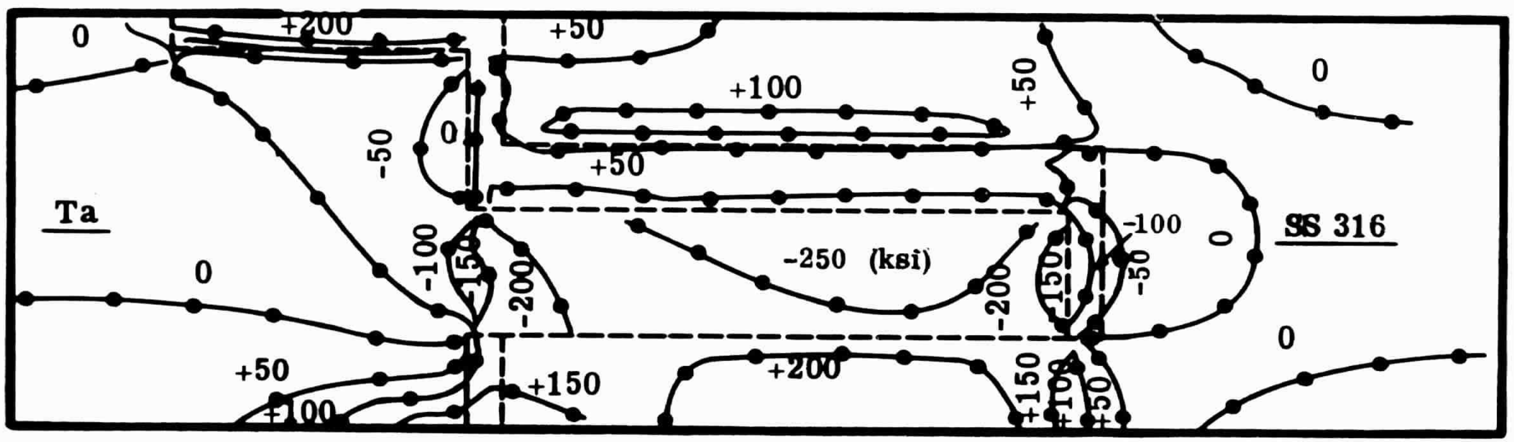


Figure 5. Iso-stress Map in Multi-material Joint Residual Axial Stresses (Elastic) Due Cool-down

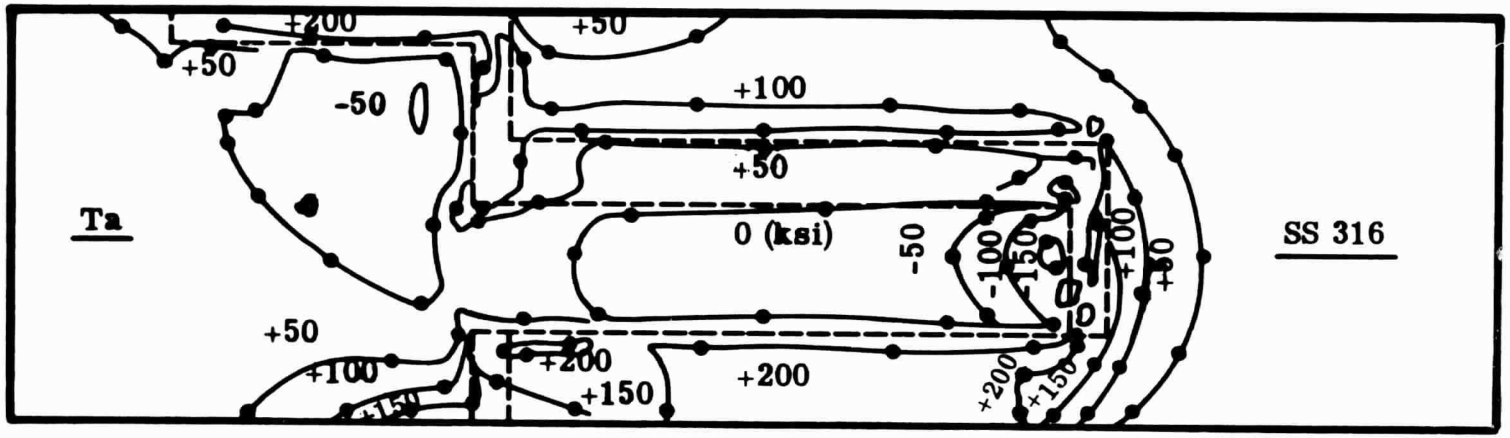


Figure 6. Iso-stress Map in Multi-material Joint Residual Maximum Principal R-Z Stresses (Elastic) Due Cool-down

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BI-LINEAR STRESS OPTION

ROW 13   S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW 12   S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW 11   S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW 10   S S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW  9   S S S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S S Y Y Y Y Y Y Y Y Y Y S S S
ROW  8   S S S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S Y Y Y Y Y Y Y Y Y Y Y S S S
ROW  7   S S S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW  6   S S S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW  5   S S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW  4   S S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW  3   S S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW  2   S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S
ROW  1   S S S S S S S Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y Y S S S

         1                 10                  20                  30                  40

368 ELEMENTS HAVE YIELDED.
```

Y = YIELDED ELEMENTS

Figure 7. Chart of Yielded Elements from Elasticity Analysis

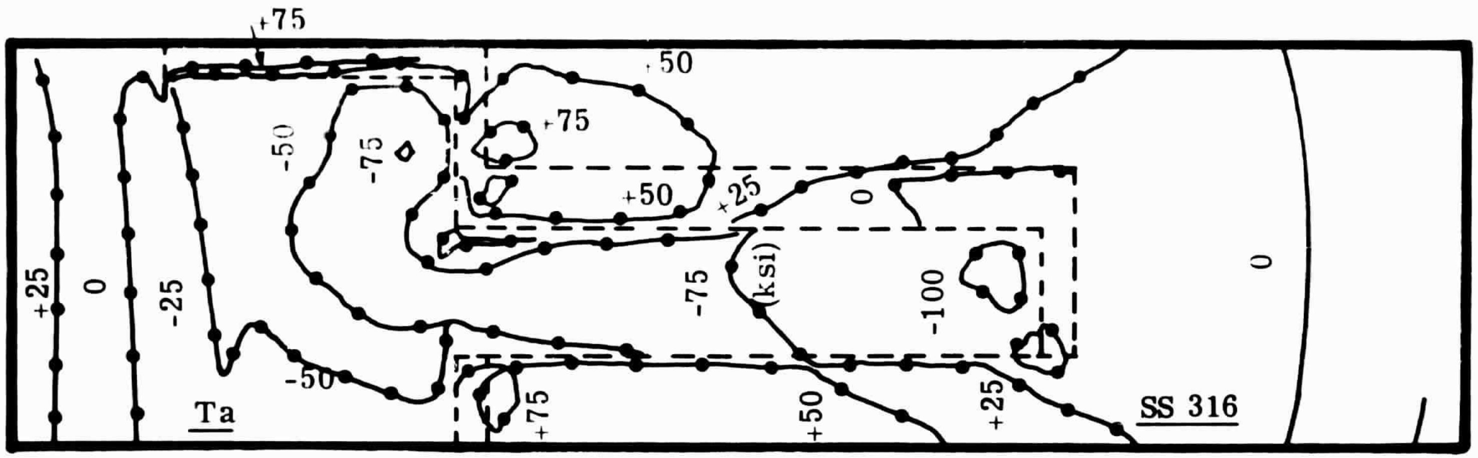

Figure 8. Residual Hoop Stresses Due Cool-down Bi-linear Iterations (1)

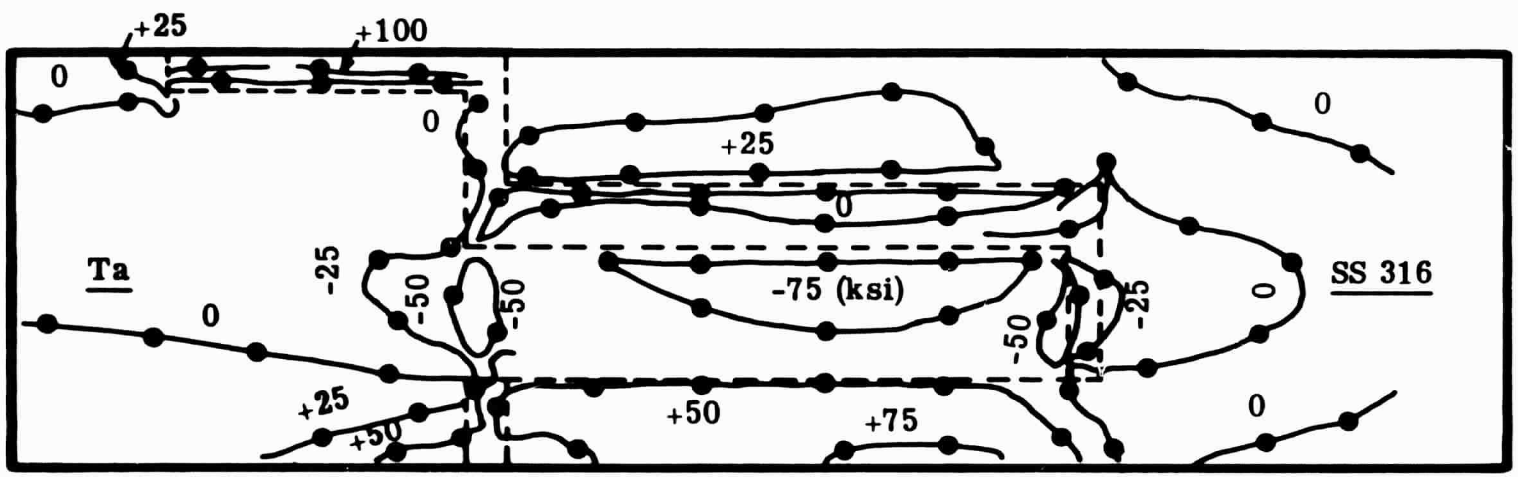

Figure 9. Residual Axial Stresses Due Cool-down Bi-linear Iterations (1)

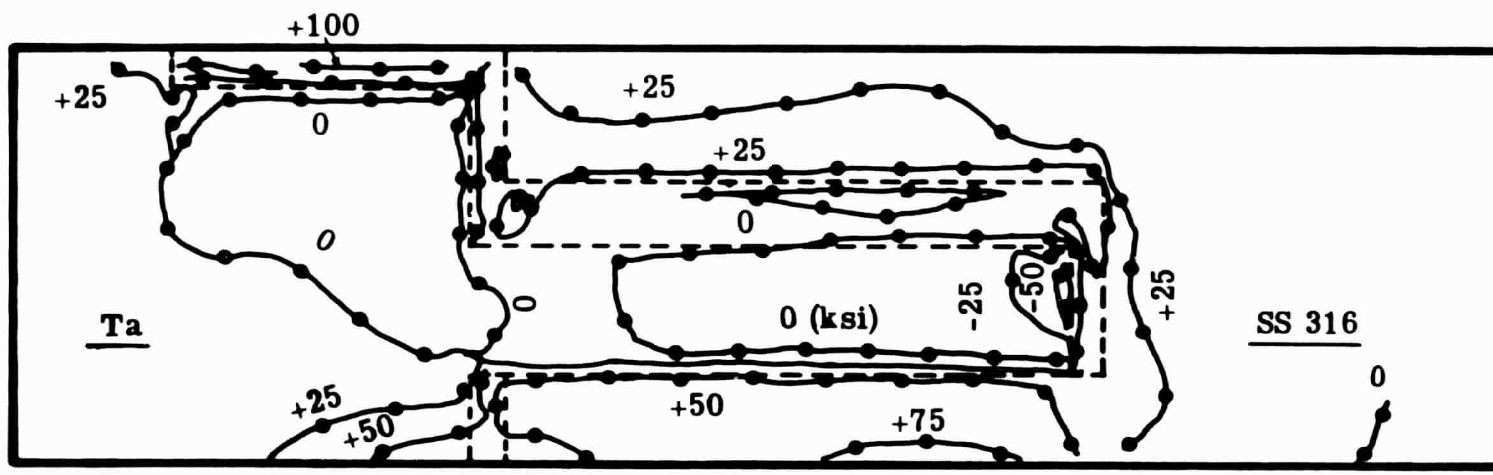

Figure 10. Residual Maximum Principal R-Z Stresses Due Cool-down Bi-linear Iteration (1)

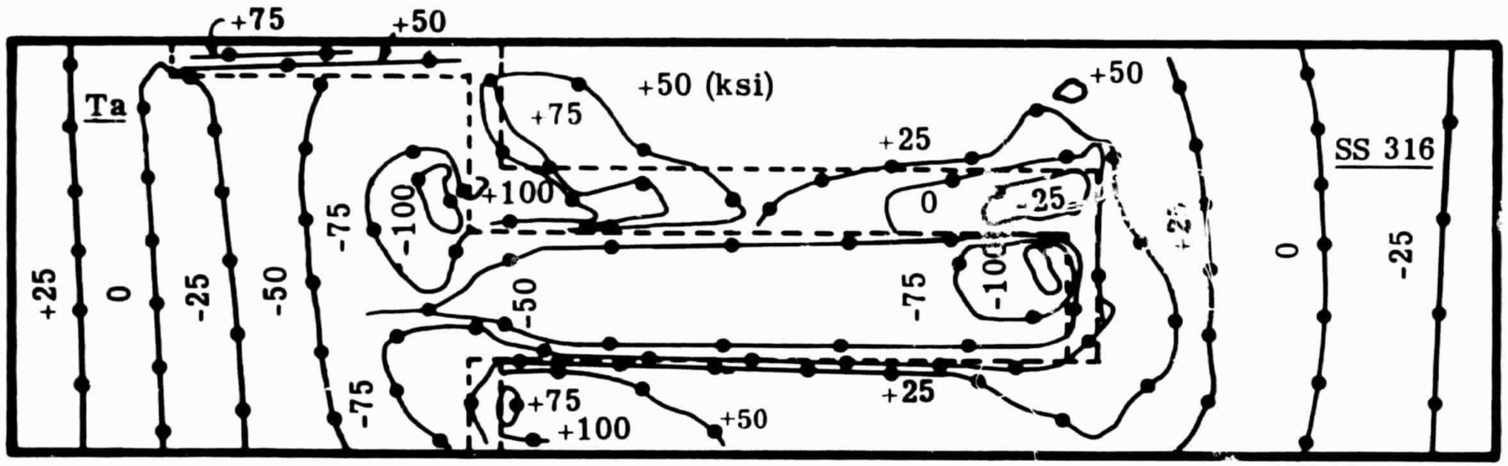

Figure 11. Residual Hoop Stresses Due Cool-down Bi-linear Iteration (2)

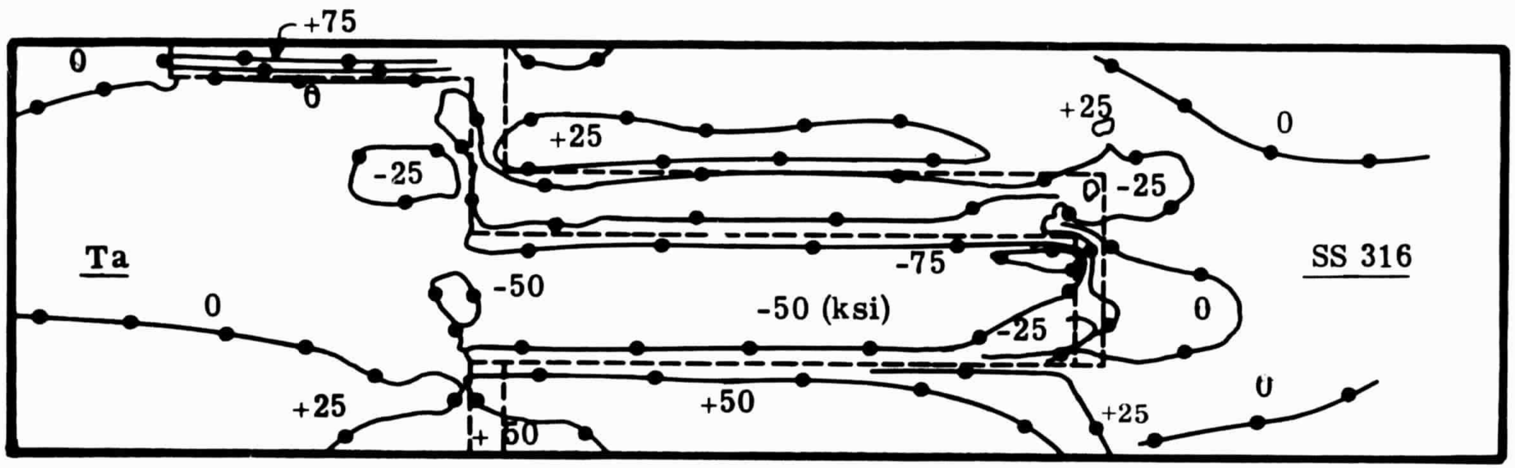

Figure 12. Residual Axial Stresses Due Cool-down Bi-linear Iteration (2)

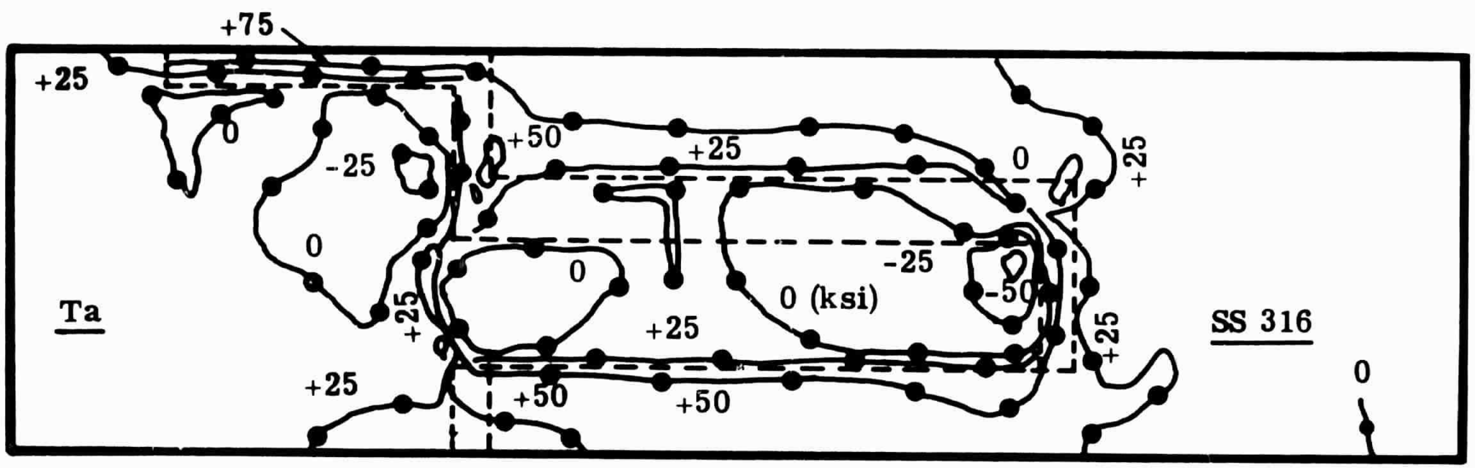

Figure 13. Residual Maximum Principal R-Z Stresses Due Cool-down Bi-linear Iteration (2)

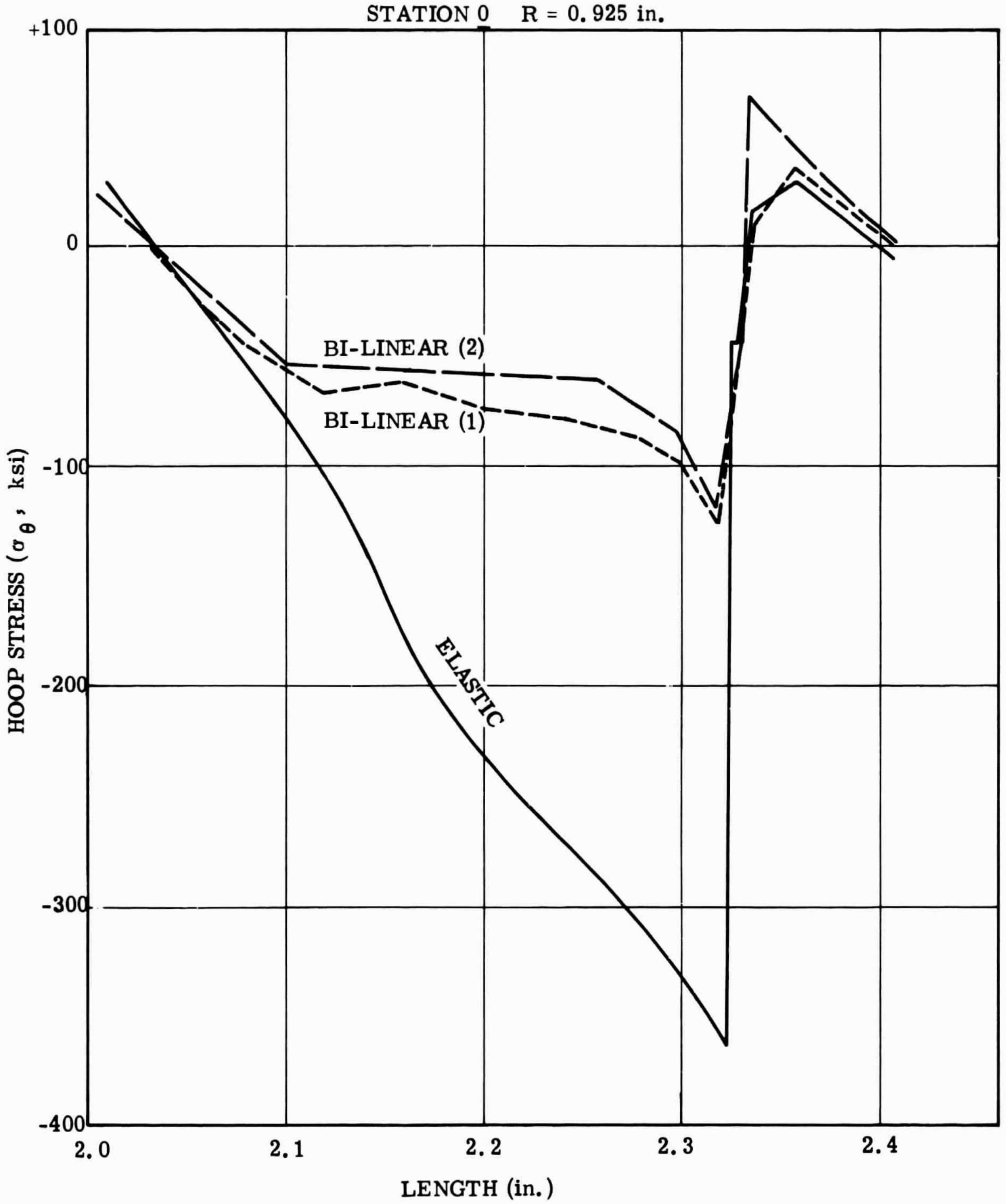

Figure 14. Lengthwise Variation of Residual Hoop Stresses

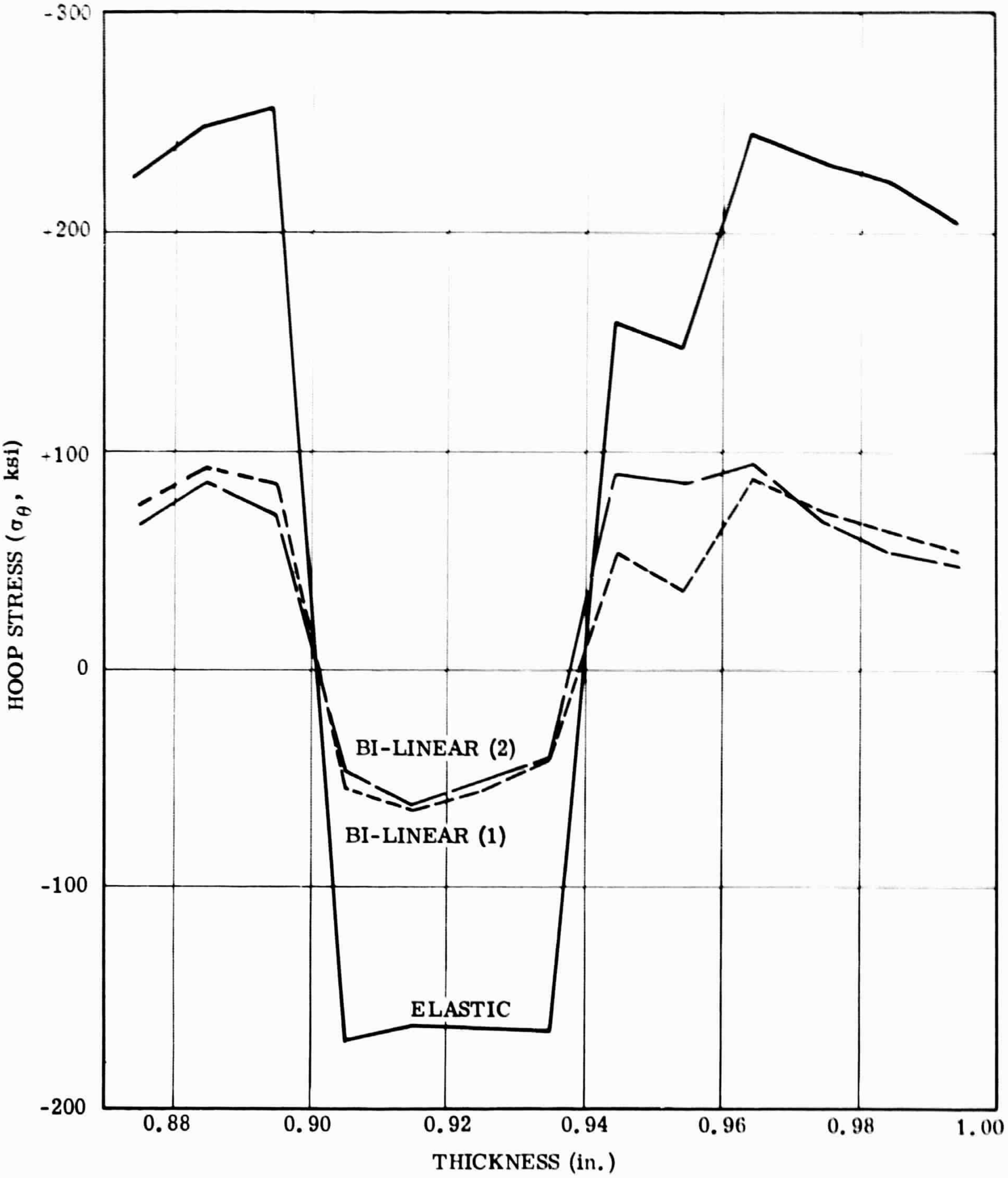

Figure 15. Variation of Residual Hoop Stresses Across Station A (Z = 2.15 in.)

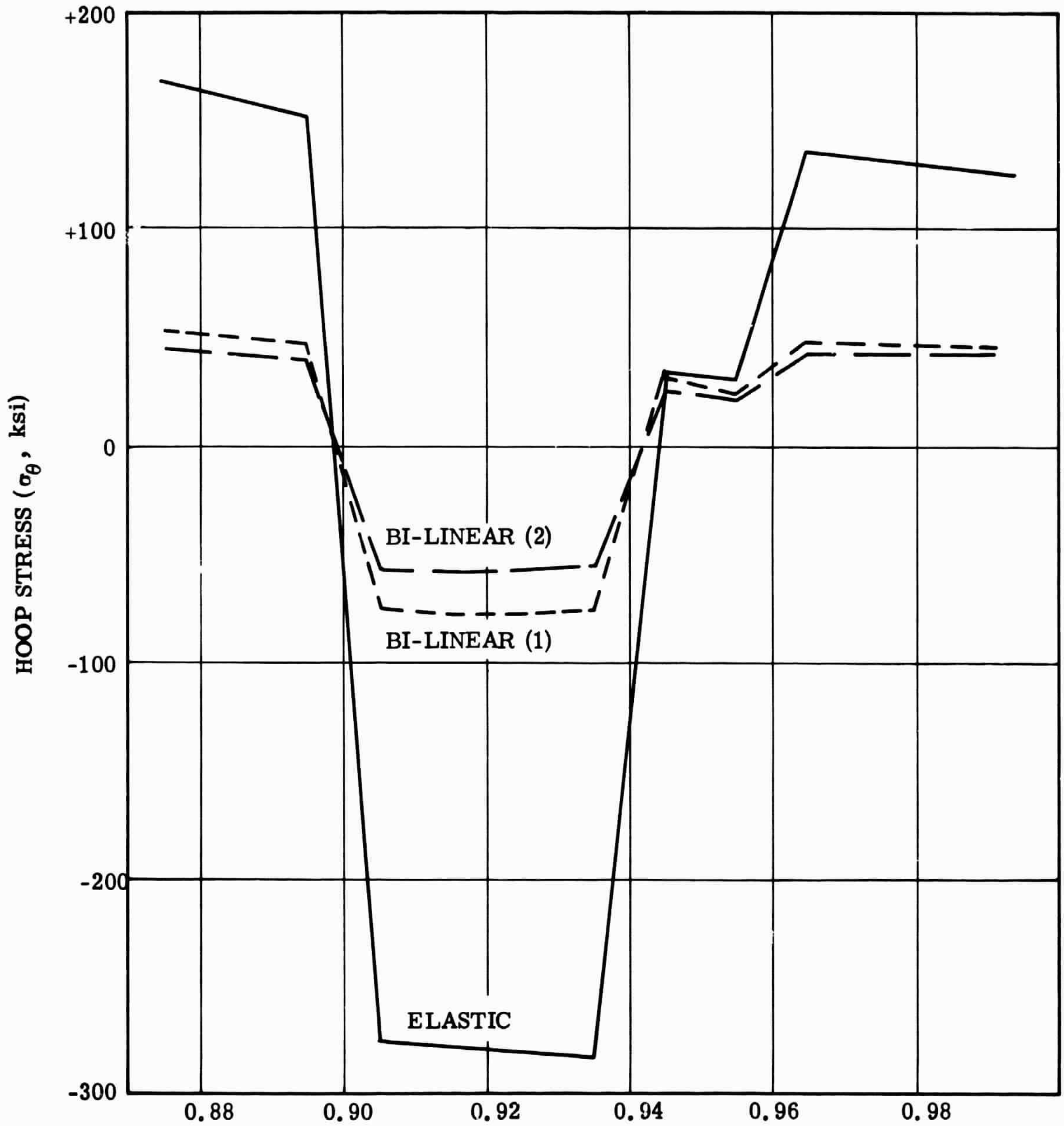

Figure 16. Variation of Residual Hoop Stresses Across Station B (Z = 2.25 in.)

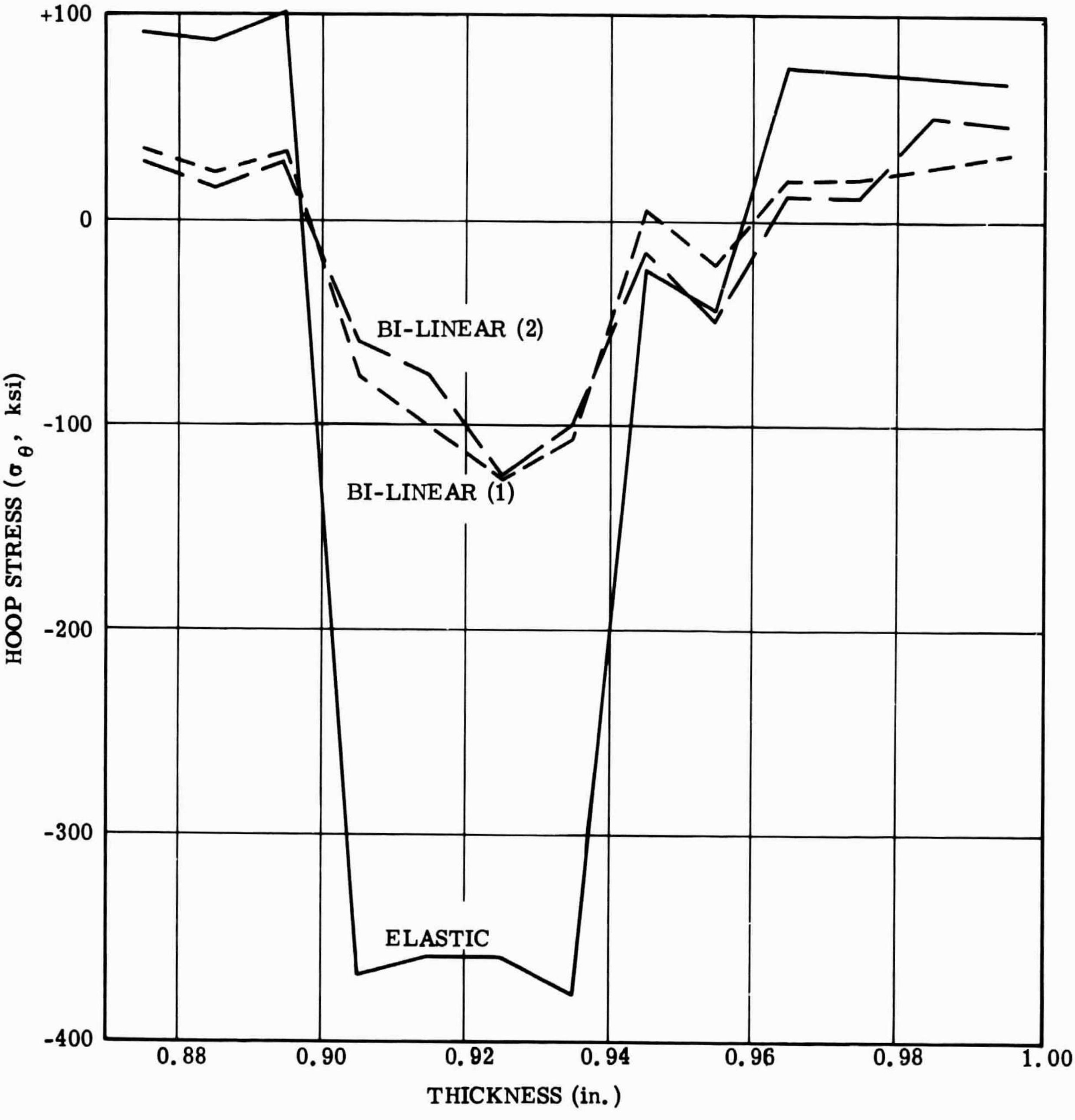

Figure 17. Variation of Residual Hoop Stresses Across Station C (Z = 2.32 in.)

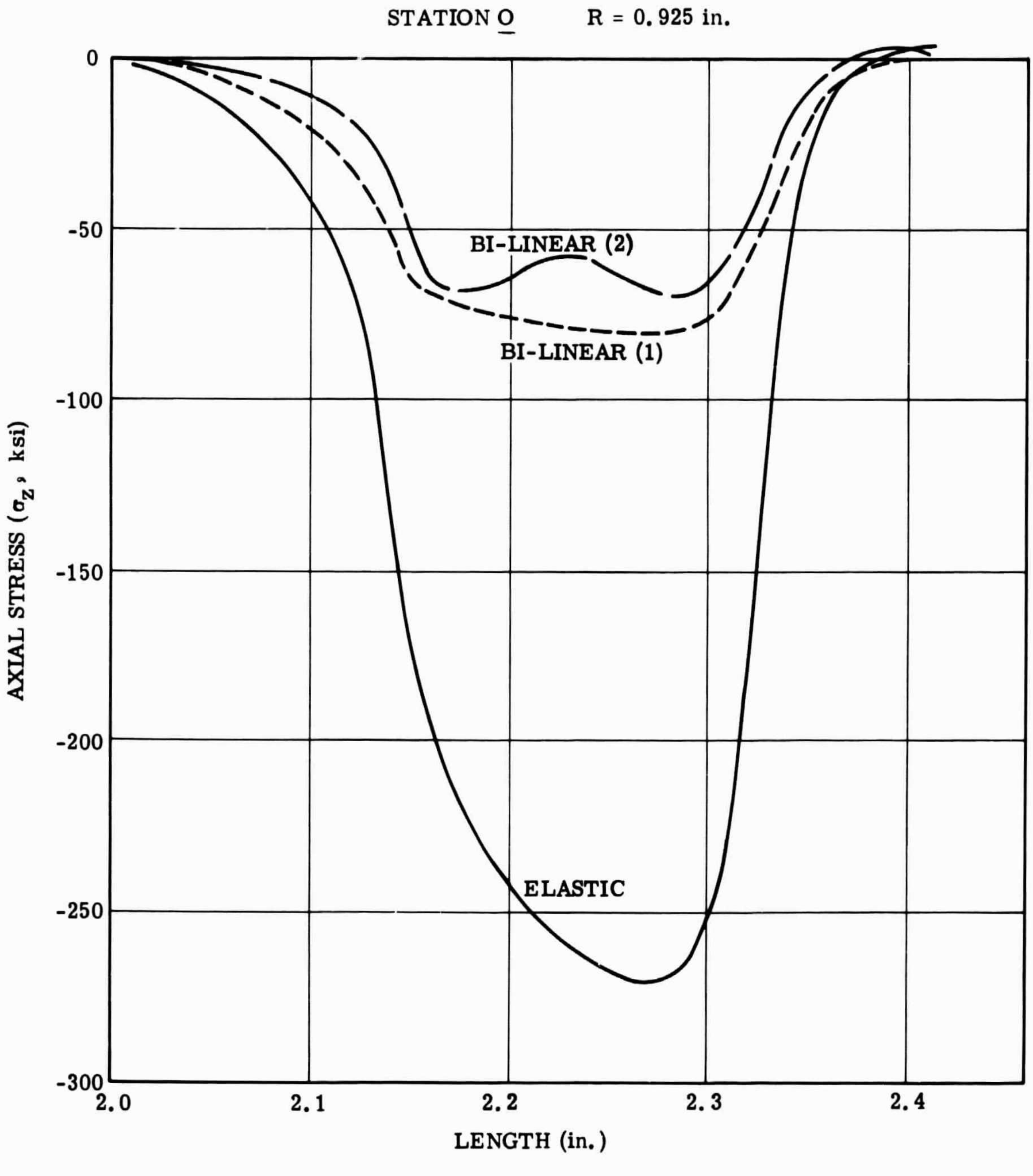

Figure 18. Lengthwise Variation of Residual Axial Stresses

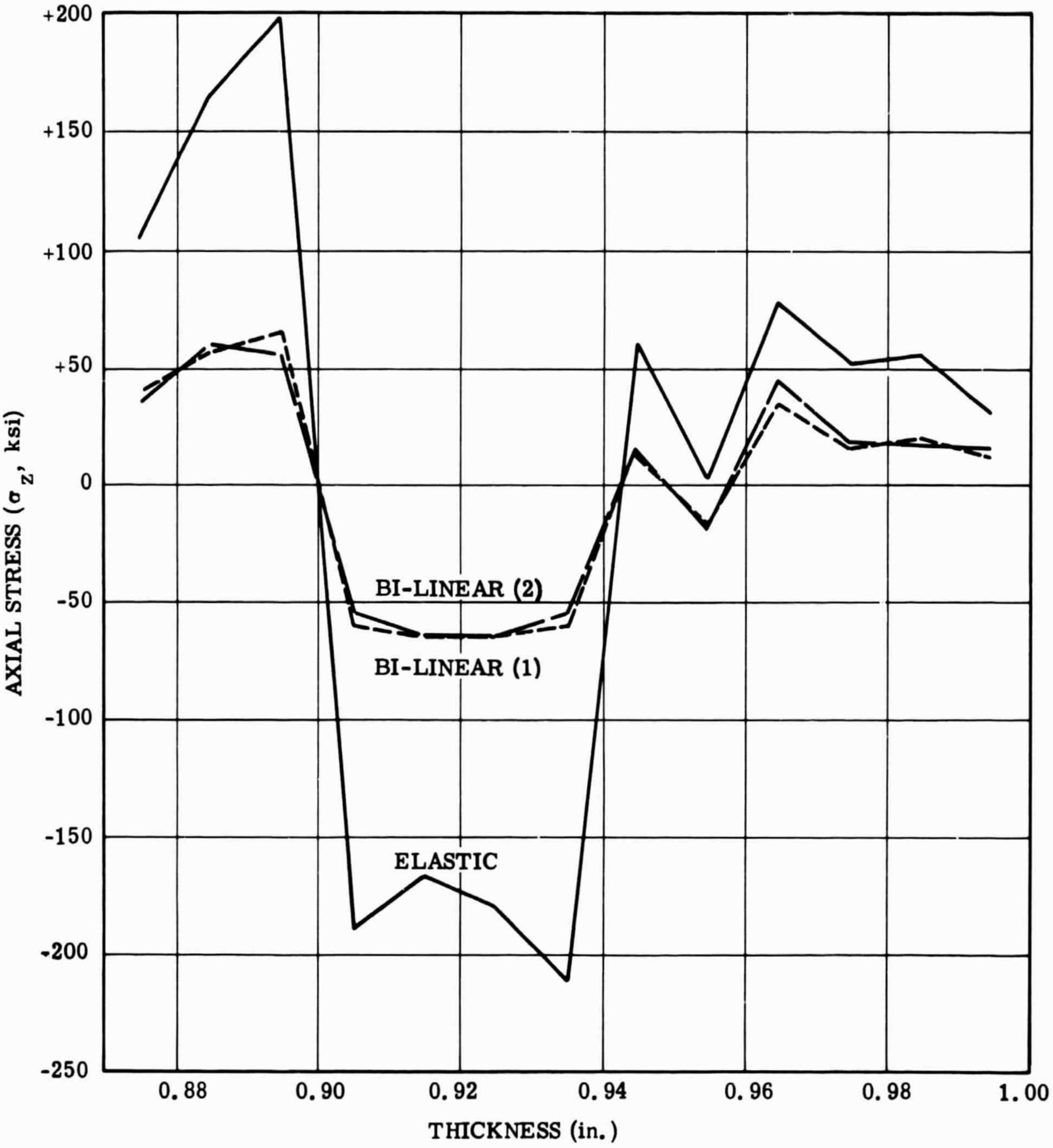

Figure 19. Variation of Residual Axial Stresses Across Station A (Z = 2.15 in.)

Figure 20. Variation of Residual Axial Stresses Across Station B (Z = 2.25 in.)

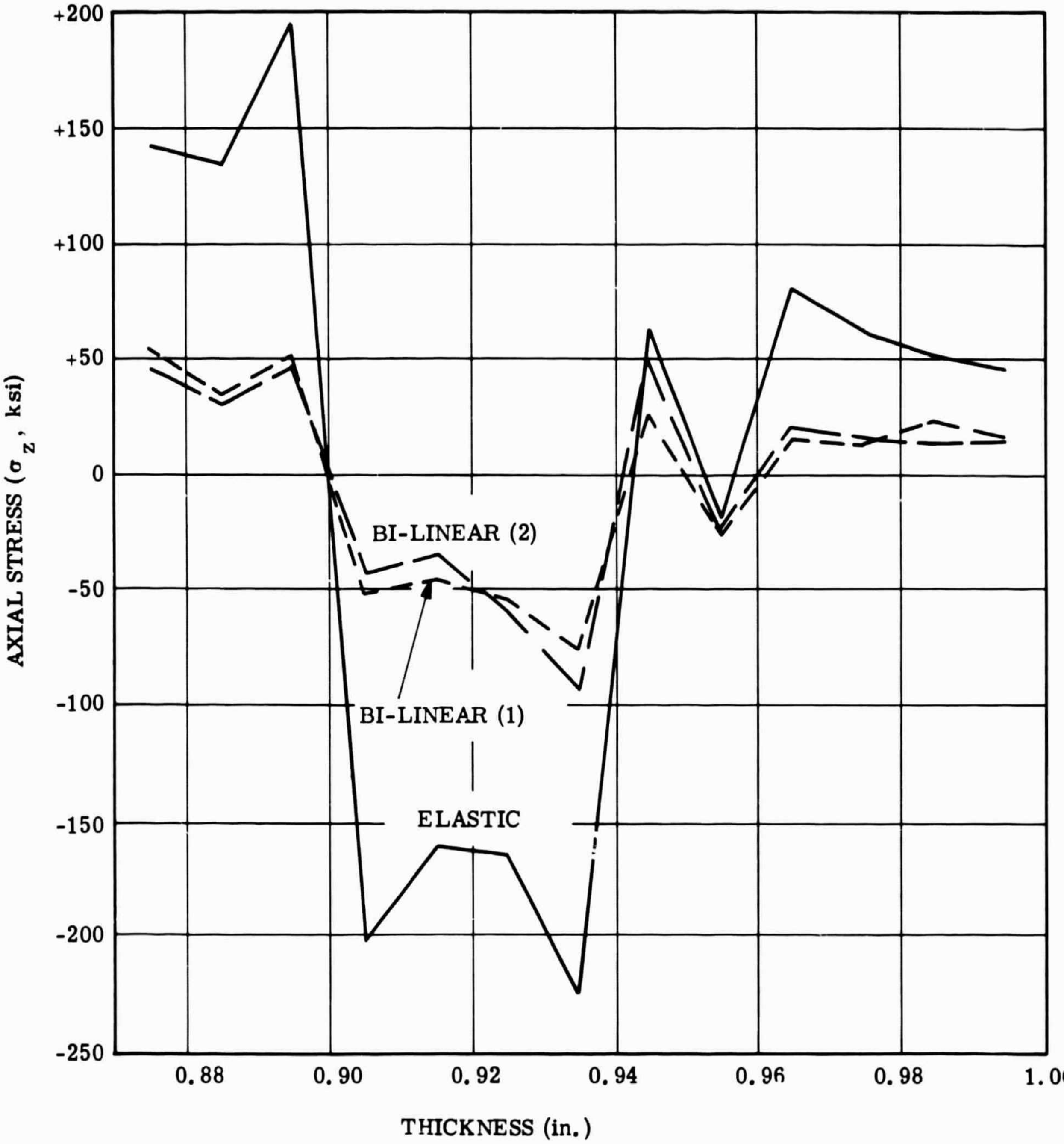

Figure 21. Variation of Residual Axial Stresses Across Station C (Z = 2.32 in.)

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7.0 ACKNOWLEDGEMENTS

The paper presented is abstracted from a study contract performed under NASA Contract NAS 3-10610 of which E. Furman is the Project Manager. The writer is grateful to R. Brooks, GE Nuclear System Programs, and A. Ross, Manager, Solid Mechanics Laboratory, for their encouragement and guidance in the course of the study.